

1. A particle P of mass 0.5 kg is attached to one end of a light elastic spring, of natural length 1.2 m and modulus of elasticity λ newtons. The other end of the spring is attached to a fixed point A on a ceiling. The particle is hanging freely in equilibrium at a distance 1.5 m vertically below A .

(a) Find the value of λ . **(3)**

The particle is now raised to the point B , where B is vertically below A and $AB = 0.8\text{ m}$. The spring remains straight. The particle is released from rest and first comes to instantaneous rest at the point C .

(b) Find the distance AC . **(4)**



2. The finite region bounded by the x -axis, the curve with equation $y = 2e^x$, the y -axis and the line $x = 1$ is rotated through one complete revolution about the x -axis to form a uniform solid.

Use algebraic integration to

- (a) show that the volume of the solid is $2\pi(e^2 - 1)$, (4)

- (b) find, in terms of e , the x coordinate of the centre of mass of the solid. (6)



3.

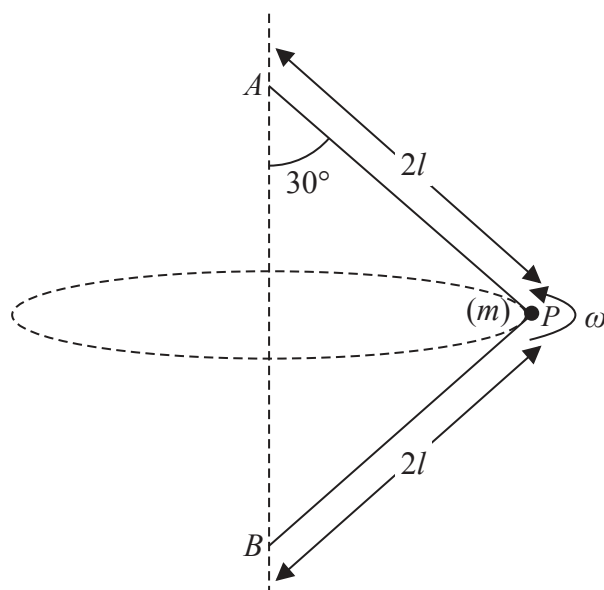


Figure 1

A small ball P of mass m is attached to the midpoint of a light inextensible string of length $4l$. The ends of the string are attached to fixed points A and B , where A is vertically above B . Both strings are taut and AP makes an angle of 30° with AB , as shown in Figure 1. The ball is moving in a horizontal circle with constant angular speed ω .

(a) Find, in terms of m , g , l and ω ,

(i) the tension in AP ,

(ii) the tension in BP .

(8)

(b) Show that $\omega^2 \geq \frac{g\sqrt{3}}{3l}$.

(2)



4. A vehicle of mass 900 kg moves along a straight horizontal road. At time t seconds the resultant force acting on the vehicle has magnitude $\frac{63000}{kt^2}$ N, where k is a positive constant. The force acts in the direction of motion of the vehicle. At time t seconds, $t \geq 1$, the speed of the vehicle is v m s⁻¹ and the vehicle is a distance x metres from a fixed point O on the road. When $t = 1$ the vehicle is at rest at O and when $t = 4$ the speed of the vehicle is 10.5 m s⁻¹.

(a) Show that $v = 14 - \frac{14}{t}$ **(7)**

(b) Hence deduce that the speed of the vehicle never reaches 14 m s⁻¹. **(1)**

(c) Use the trapezium rule, with 4 equal intervals, to estimate the value of x when $v = 7$ **(4)**



6.

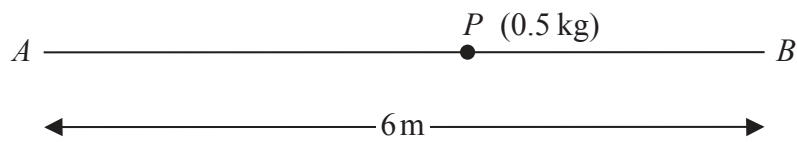


Figure 3

Two points A and B are 6 m apart on a smooth horizontal floor. A particle P of mass 0.5 kg is attached to one end of a light elastic spring, of natural length 2.5 m and modulus of elasticity 20 N . The other end of the spring is attached to A . A second light elastic spring, of natural length 1.5 m and modulus of elasticity 18 N , has one end attached to P and the other end attached to B , as shown in Figure 3. Initially P rests in equilibrium at the point O , where AOB is a straight line.

(a) Find the length of AO .

(4)

The particle P now receives an impulse of magnitude 6 N s acting in the direction OB and P starts to move towards B .

(b) Show that P moves with simple harmonic motion about O .

(4)

(c) Find the amplitude of the motion.

(4)

(d) Find the time taken by P to travel 1.2 m from O .

(3)



7. A solid smooth sphere, with centre O and radius r , is fixed to a point A on a horizontal floor. A particle P is placed on the surface of the sphere at the point B , where B is vertically above A . The particle is projected horizontally from B with speed $\frac{\sqrt{gr}}{2}$ and starts to move on the surface of the sphere. When OP makes an angle θ with the upward vertical and P remains in contact with the sphere, the speed of P is v .

(a) Show that $v^2 = \frac{gr}{4} (9 - 8 \cos \theta)$. (4)

The particle leaves the surface of the sphere when $\theta = \alpha$.

(b) Find the value of $\cos \alpha$. (4)

After leaving the surface of the sphere, P moves freely under gravity and hits the floor at the point C .

Given that $r = 0.5$ m,

(c) find, to 2 significant figures, the distance AC . (7)



Question 7 continued

Lined area for writing the answer to Question 7.



